# Turbulent Properties of a Flat-Plate Boundary Layer with a Diffusion Flame

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**An experimental investigation on a flat-plate turbulent boundary layer with mass addition from a porous plate and a diffusion flame has been made. The mean velocity and turbulent intensity of a fluctuating velocity parallel to the freestreamwise direction ($\bar{U}$ and $u'$) and a correlation of fluctuating velocities parallel and normal to the freestreamwise direction ($\overline{uv}$) have been measured using a laser Doppler velocimeter. A hydrogen/nitrogen mixture was injected as a fuel from a porous plate. The freestream velocity was 10 m/s and the blowing mass ratio of a mass injection and free airstream was 0.010. The profile of $-\overline{uv}$ corresponds to the profile of the mean velocity gradient normal to the freestreamwise direction in the whole region except near the flame zone. These results suggest that the turbulence is an ordinary shear-generated turbulence in the whole region of the boundary layer except near the flame zone. Near the flame zone, an influence of nonordinary shear-generated turbulence on the profile of $-\overline{uv}$ is expected.**

## Nomenclature

| | |
|---|---|
| $N$ | = number of data points |
| $T$ | = temperature, °C |
| $U$ | = velocity in $x$ direction, m/s |
| $U_i$ | = individual velocity in $x$ direction, m/s |
| $u$ | = fluctuating velocity in $x$ direction, m/s |
| $u'$ | = turbulent intensity of fluctuating velocity in $x$ direction, m/s |
| $\overline{uv}$ | = correlation of fluctuating velocities in $x$ and $y$ directions, $m^2/s^2$ |
| $V$ | = velocity in $y$ direction, m/s |
| $v'$ | = turbulent intensity of fluctuating velocity in $y$ direction, m/s |
| $x$ | = coordinate parallel to the freestreamwise direction as defined in Fig. 1, mm |
| $y$ | = coordinate normal to the freestreamwise direction as defined in Fig. 1, mm |
| $\beta$ | = blowing ratio |
| $\Delta$ | = deviation of the statistical value |
| $\delta$ | = velocity boundary-layer thickness, mm |
| $\epsilon$ | = arbitrary constant of data reduction method |
| $\rho$ | = density, $kg/m^3$ |
| $\sigma$ | = standard deviation |

*Superscript*

$(\bar{\ })$ = mean value

*Subscript*

$\infty$ = freestream

## Introduction

THE fluid mechanical and thermal structure of a flat-plate turbulent boundary layer with mass addition and a diffusion flame has been investigated in connection with various applications such as ablative cooling, erosive burning, and the surface combustion of liquid fuels.[1-6] Measurements of mean velocity, turbulent intensity of fluctuating velocity, mean temperature, and turbulent intensity of fluctuating temperature have been made by the authors.[5] The measured data show that the mean velocity profile reaches a maximum in the free airstream side of the flame and that the turbulent intensity of a fluctuating velocity at the flame zone increases, unlike that of the isothermal boundary-layer flow.

Detailed discussions on turbulent properties of jet diffusion flames have been made.[7,8] Glass and Bilger[7] elucidated that the profiles of mean velocity, the turbulent intensity of a fluctuating velocity, and a correlation of fluctuating velocities parallel and normal to a streamwise direction were essentially the same as those of the isothermal flow in the case of a turbulent jet diffusion flame. Takagi et al.[8] pointed out that when the flame was formed, the turbulent intensity of a fluctuating velocity decreased, unlike that of the isothermal flow, while the profile of the correlation was essentially the same as that of the isothermal flow. As pointed out by Bilger,[9] these results suggest that, in the case of a turbulent jet diffusion flame, the turbulence is an ordinary shear-generated turbulence and the flowfield with a diffusion flame may be predicted with good accuracy using the same turbulent models as used for predicting nonburning flow.

In recent years, a numerical calculation of a flat-plate turbulent boundary layer with a diffusion flame has been made.[4] Because of the lack of useful data on turbulence modeling, a turbulence model used for predicting nonburning flow has been applied. In order to develop the method of theoretical prediction, useful data on turbulent modeling are required. The profile of the correlation of the fluctuating velocity parallel and normal to the freestreamwise direction ($\overline{uv}$) is one of the important properties, as well as profiles of the mean velocity and the turbulent intensity of fluctuating velocity in the freestreamwise direction ($\bar{U}$ and $u'$) to elucidate the turbulence structure of a flat-plate turbulent boundary layer with a diffusion flame. For turbulence modeling, the relationship between $-\overline{uv}$ and the mean velocity gradient normal to the freestreamwise direction ($\partial U/\partial y$) is very important because it suggests the applicability of nonburning turbulent models for the prediction of the burning flow.

Thus, the purpose of the present study is to measure the profile of a correlation of fluctuating velocities parallel and normal to the freestreamwise direction in detail, to discuss the relationship between this correlation and the mean velocity gradient normal to the freestreamwise direction, and to elucidate the effect of a diffusion flame on the profile of a correlation of a flat-plate turbulent boundary layer with mass addition and diffusion flame.

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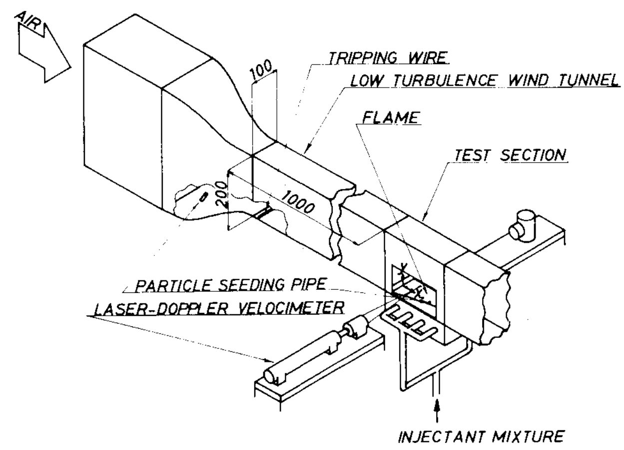


**Fig. 1 Experimental apparatus and measuring system.**

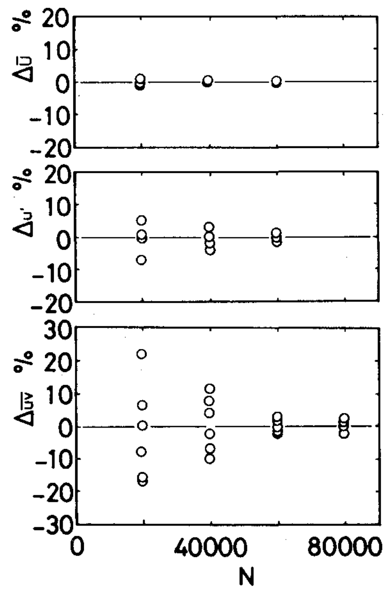


**Fig. 2 Relationship between statistical uncertainty and number of data points.**

## Experimental Apparatus and Procedure

### Wind Tunnel

The present series of experiments utilized the same low-turbulence wind tunnel previously described in detail in Ref. 5. Figure 1 is a schematic of the wind tunnel and the measuring system.

The turbulent intensity of a fluctuating velocity in a freestream $[(u'/\bar{U})_\infty]$ is about 0.7%. The test section is $100 \times 200$ mm in cross section. A tripping wire is mounted on a bottom wall just downstream of the nozzle exit to obtain a fully developed isothermal nontranspired turbulent boundary layer at the test section. A porous plate of sintered stainless steel $96 \times 200$ mm in surface area and 5 mm in thickness is mounted flush with the bottom wall of the test section. For combustion experiments, a hydrogen/nitrogen mixture with the mass concentration of 4% hydrogen was uniformly injected through the porous plate. For isothermal experiments with mass addition, air was injected through the porous plate. The freestream velocity was kept constant at about 10 m/s. The coordinate system is defined as shown in Fig. 1. In the present study, measurements were made at two $x$ positions (150 and 180 mm) to elucidate the variation in the profiles with the distance from the leading flame edge. The blowing mass ratio $[\beta = (\rho \bar{V})_{\text{injectant gas}}/(\rho \bar{U})_\infty]$ was kept constant at 0.010.

### Velocity Measurement

A dual-beam laser Doppler velocimeter (LDV) was used to collect the velocity data. The LDV signals were processed by a counter-type signal processor interfaced with a digital microcomputer. $TiO_2$ was used as scattering particles. In the data stored in the computer memories, infrequent but not negligible noisy data were included. In the present series of experiments, the $\epsilon$-$\sigma$ method ($\epsilon = 6$) was used to eliminate the noisy data. (The details of the $\epsilon$-$\sigma$ method are described in Ref. 5.) The remaining data were used to calculate $\bar{U}$, $u'$, and $\overline{uv}$. The mean velocity and turbulent intensity of a fluctuating velocity in the freestreamwise direction ($\bar{U}$ and $u'$) are calculated from direct measured data, using the following equation:

$$\bar{U} = \sum_{i=1}^{N} U_i / N \tag{1}$$

$$u' = \left[ \sum_{i=1}^{N} (U_i - \bar{U})^2 / N \right]^{1/2} \tag{2}$$

where $U_i$ is the individual velocity data and $N$ the number of data points. The correlation $(\overline{uv})$ is calculated by

$$\overline{uv} = (u_+'^2 - u_-'^2)/2 \tag{3}$$

where $u_+'$ and $u_-'$ are the turbulent intensities of the fluctuating velocities in +45 and −45 deg inclined directions to the freestreamwise direction, respectively.

Scattering particles were seeded into the airstream primarily from a pipe (i.d. = 3 mm) at the nozzle section. They were also seeded into the injectant gas stream from six pipes mounted in the porous plate, since few particles seeded in the airstream dispersed into the region between the flame and the porous plate. They were not seeded simultaneously. That is, measurements were made by scattering particles seeded from either the pipe at the nozzle section or the six pipes mounted in the porous plate.

Figure 2 shows dependence of statistical uncertainty on number of data points, where $\Delta_x$ is defined as

$$\Delta_x = (X_N - X_{\max}) / X_{\max} \tag{4}$$

where $X_N$ is the statistical value calculated by $N$ data and $X_{\max}$ the statistical value calculated by maximum number of data points. For $\bar{U}$ and $u'$, the maximum is 60,000 and for $\overline{uv}$ 80,000.

For $\bar{U}$ and $u'$ 40,000 data points are enough to yield reliable statistical values, while for $\overline{uv}$ more than 60,000 are necessary. Based on these results, more than 60,000 data points were used to calculate $\bar{U}$, $u'$, and $\overline{uv}$ in one measurement in the present study.

### Temperature Measurement

The profile of mean temperature $\bar{T}$ was measured with a 50 μm Pt/Pt13%Rh thermocouple. The measured values were not corrected for the thermometric errors due to radiation, conduction, and catalytic activity.

## Results and Discussion

### Profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ Without Flame

Figure 3 shows the mean velocity profiles of the isothermal flows with and without mass injection along with the mean velocity profile of Cole's wake law.[10] The velocity profile with mass injection was obtained with the equation proposed by Stevenson.[11] Figure 4 shows the profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ without mass injection along with the profiles measured by Klebanoff.[12] Figure 5 shows the profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ with air injection along with the profiles measured by

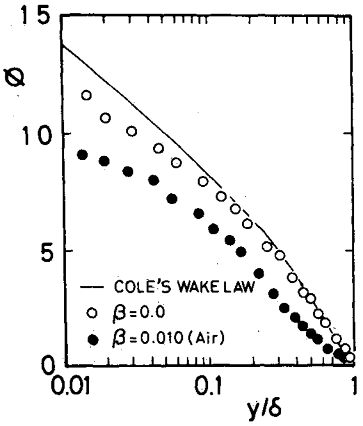


Fig. 3 Mean velocity profiles of the isothermal flow along with Cole's wake law profile.

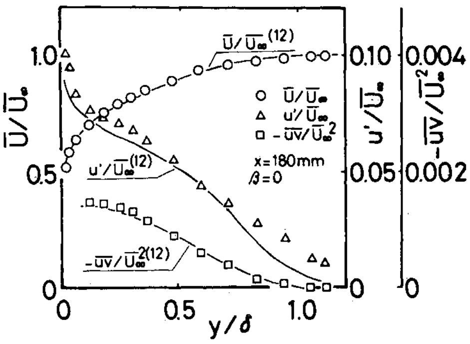


Fig. 4 Profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ at $x = 180$ mm without mass injection.

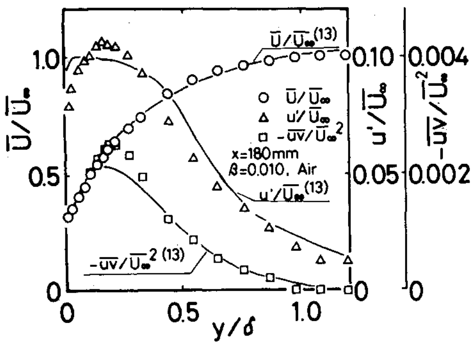


Fig. 5 Profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ at $x = 180$ mm with air injection.

Peterson and Isaacson,[13] although the experimental conditions of the present study ($Re_x = 1.2 \times 10^5$, $\beta = 0.010$) are not exactly the same as those of Peterson and Isaacson ($Re_x = 8.95 \times 10^5$, $\beta = 0.00545$). The mean velocity profile without mass injection is almost the same as that of Cole's wake law as shown in Fig. 3 and the profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ are in good agreement with those measured by Klebanoff as shown in Fig. 4. This indicates the high accuracy of the LDV system.

The mean velocity profile with mass injection is similar to that measured by Peterson and Isaacson[13] as shown in Fig. 5, although it does not coincide with Cole's wake law as shown in Fig. 3. The profile of $u'$ is also similar to that measured by Peterson and Isaacson.[13] The profile of $-\overline{uv}$ has a peak near the wall. The values of $-\overline{uv}$ are all positive in the whole region of the boundary layer and the value of the peak is larger than the maximum value of $-\overline{uv}$ without mass injection. The profile of $-\overline{uv}$ is also similar to that measured by Peterson and Isaacson.[13] These results indicate that the qualitative characteristics of profiles with mass injection are the same as those of the previous work,[13] although the boundary layer does not fully develop.

### Profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ with Diffusion Flame

Figures 6 and 7 show profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ at $x = 150$ and 180 mm, respectively. In Figs. 6 and 7, ○ indicates values of $\bar{U}$, $u'$, and $-\overline{uv}$ measured with "airstream seeding particles" and ● those measured with "injectant gas seeding particles." The solid lines represent the reliable profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ resulting from the consideration mentioned in Ref. 5. Typical characteristics of $\bar{U}$ and $u'$ are that the peak of $\bar{U}$ is observed in the airstream side of the flame and the peak of $u'$ is observed at the flame zone, as discussed in detail in Ref. 5.

As the flame was formed in the boundary layer, the main flow was slightly accelerated from 10.0 m/s at $x = 120$ mm to 10.3 m/s at $x = 180$ mm. Then, the static pressure gradient in the main flow between $x = 120$ and 180 mm is estimated to be about $-61$ Pa/m. The effect of the static pressure gradient in the main flow on the fluid dynamical structure is discussed in detail in Ref. 2. The results in Ref. 2 show that the mean velocity profile with a static pressure gradient in the main flow is qualitatively the same with that with a zero pressure gradient, although the profiles vary slightly quantitatively. These indicate that the effect of static pressure gradient in the main flow on the fluid dynamical structure can be considered to be negligible in the present study.

As shown in Figs. 6a and 7a, since a minimum and a maximum exist in the profile of $\bar{U}$, the sign of the mean velocity gradient ($\partial\bar{U}/\partial y$) changes at those points. In the region from the wall to the maximum, the sign of $\partial\bar{U}/\partial y$ is positive. In the region from the maximum to the minimum, the sign of $\partial\bar{U}/\partial y$ is negative. From the minimum to the boundary-layer edge, the sign of $\partial\bar{U}/\partial y$ becomes positive once more. As is well known, the value of gradients at the maximum and minimum is zero. Furthermore, it is noteworthy that the gradient becomes maximum near the flame zone.

Figures 6b and 7b show the profiles of $-\overline{uv}$ at $x = 150$ and 180 mm, respectively. The value of $-\overline{uv}$ close to the wall ($y/\delta \lesssim 0.1$) can not be measured since the LDV beam is inclined $\pm 45$ deg to the freestreamwise direction. In the freestream region ($y/\delta > 1.0$), $-\overline{uv}$ becomes zero as expected. As $y/\delta$ is decreased from the boundary-layer edge, the value of $-\overline{uv}$ increases. This tendency is quite similar to that of the isothermal flow with mass injection, which is shown in Figs. 6b and 7b. In the case of the isothermal flow with mass injection, the value of $-\overline{uv}$ continues to increase as $y/\delta$ decreases to $y/\delta = 0.3$. On the other hand, the profile of $-\overline{uv}$ with a diffusion flame reaches the freestream side maximum in the middle region of the boundary layer ($y/\delta \doteq 0.5 \sim 0.6$) and begins to decrease with further decrease in $y/\delta$. With further decrease in $y/\delta$, the value of $-\overline{uv}$ becomes negative, crossing the zero point at $y/\delta \doteq 0.45$, and reaches minimum at $y/\delta \doteq 0.35$. The value of $-\overline{uv}$ again increases as $y/\delta$ is decreased from the position of minimum and becomes positive, crossing the zero point and reaching wall side maximum at $y/\delta \doteq 0.2$. The value of $-\overline{uv}$ decreases as $y/\delta$ is decreased from the wall side maximum to the wall. Then, typical characteristics are summarized as follows:

1) The profile of $-\overline{uv}$ has two maxima and one minimum.

2) The value of $-\overline{uv}$ becomes negative in the airstream side of the flame zone.

### Comparison between Profiles of $-\overline{uv}$ and $\partial\bar{U}/\partial y$

Figures 8 and 9 show profiles of $-\overline{uv}/\bar{U}_\infty^2$ and those of nondimensional mean velocity gradients normal to the freestream direction $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ at $x = 150$ and 180 mm,

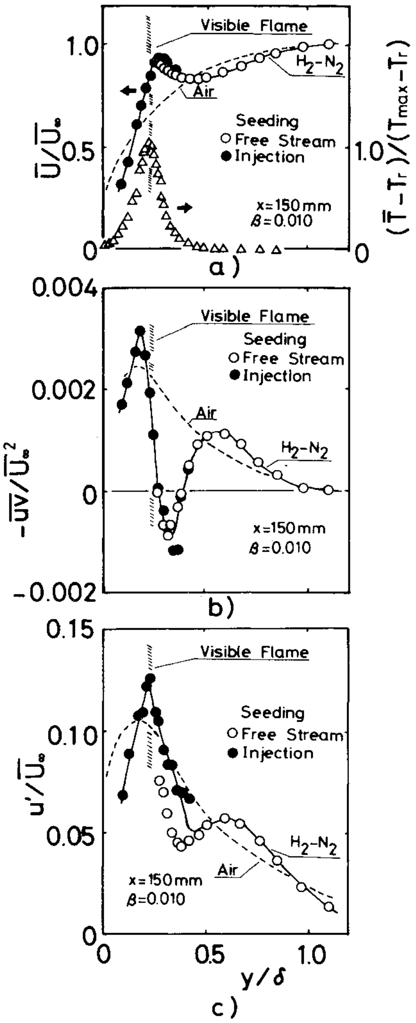


**Fig. 6 Profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ at $x = 150$ mm with diffusion flame.**

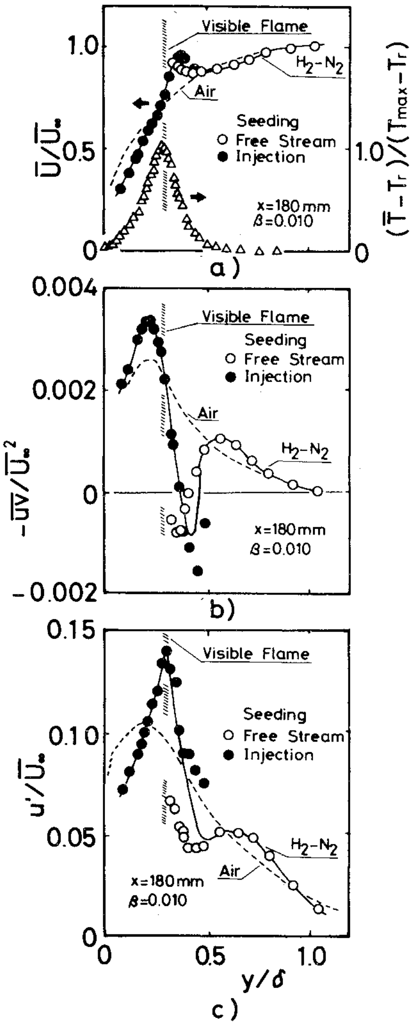


**Fig. 7 Profiles of $\bar{U}$, $u'$, and $-\overline{uv}$ at $x = 180$ mm with diffusion flame.**

respectively. The value of $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ was obtained graphically from the mean velocity profiles, which are shown in Figs. 6a and 7a as solid lines. In Figs. 8b and 9b, values of $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ involving uncertainty are shown as a shaded band.

At $x = 150$ mm, profiles of $-\overline{uv}/\bar{U}_\infty^2$ and $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ are similar to each other as shown in Fig. 8. The value of $-\overline{uv}/\bar{U}_\infty^2$ becomes zero at the location where the value of $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ becomes zero. The signs of $-\overline{uv}/\bar{U}_\infty^2$ and $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ at each $y$ position are the same in the whole region of the boundary layer. Furthermore, the locations of $y$ of the two maxima and one minimum of $-\overline{uv}/\bar{U}_\infty^2$ approximately coincide with those of the maxima and minimum of $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$. That is, the variation of $-\overline{uv}/\bar{U}_\infty^2$ qualitatively corresponds to the variation of $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$. Then, the results mentioned above reveal that the correlation $(-\overline{uv})$ is closely associated with the mean velocity gradient even in a reacting turbulent boundary layer. These results are consistent with those of the turbulent jet diffusion flame mentioned in Refs. 7-9.

In the case of a reacting turbulent boundary layer over a flat plate, some factors should be taken into account. One is the variation in properties such as density and viscosity with variations in temperature and concentration of components. Of course, this problem is always important when treating reacting flows. The other factor is that the value of $\partial\bar{U}/\partial y$ becomes zero at some locations due to local acceleration. The same problem occurs in some special conditions of a nonreacting boundary layer, e.g., film cooling processes, and some special treatment must be considered when a turbulence model is considered.[14] Although the factors mentioned above should be taken into account, the turbulence can be treated as an ordinary shear-generated turbulence, since the experimental results mentioned above show that the correlation $(-\overline{uv})$ is associated with the mean velocity gradient in a reacting turbulent boundary layer as well as that in an isothermal turbulent boundary layer.

At $x = 180$ mm, the profiles of $-\overline{uv}/\bar{U}_\infty^2$ and $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ are also similar, as shown in Fig. 8. Locations of the zero crossing and the signs of the values of $-\overline{uv}/\bar{U}_\infty^2$ and $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ coincide in the whole region of the boundary layer. At the flame zone, however, a sharp maximum peak of $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ appears, as shown in Fig. 9b, although the corresponding sharp peak is not observed in the profile at $x = 150$ mm (see Fig. 8b). On the other hand, the profile of $-\overline{uv}/\bar{U}_\infty^2$ has a peak in a wall side region of the flame zone similar to the profile of $x = 150$ mm. Thus, the wall side maximum peak of $-\overline{uv}/\bar{U}_\infty^2$ does not coincide with the peak of $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$; that is, the variation of $-\overline{uv}/\bar{U}_\infty^2$ does not correspond to the variation of $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ directly near the flame zone. Thus, the correlation $(-\overline{uv})$ is not simply associated with the mean velocity gradient near the flame zone. This suggests that a nonordinary shear-generated turbulence plays an important role at the flame zone in the downstream region.

At the flame zone, chemical reaction rate fluctuates since the concentrations of fuel and oxygen are supposed to be fluctuated by a turbulence mixing. This fluctuation of the

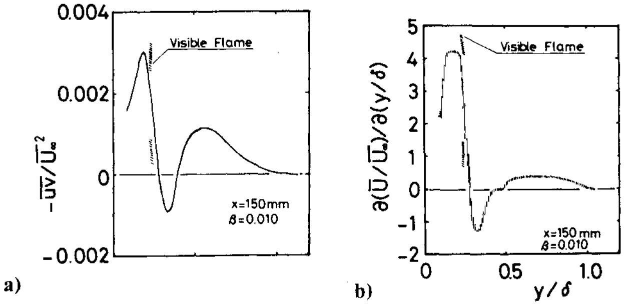


**Fig. 8 Comparison between $-\overline{uv}/\bar{U}_\infty^2$ and $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ at $x = 150$ mm with diffusion flame.**

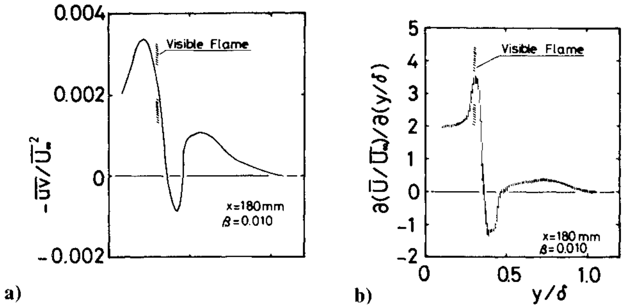


**Fig. 9 Comparison between $-\overline{uv}/\bar{U}_\infty^2$ and $\partial(\bar{U}/\bar{U}_\infty)/\partial(y/\delta)$ at $x = 180$ mm with diffusion flame.**

chemical reaction rate appears to vary the fluctuation of the fluid parameters, such as density, pressure, and/or velocity. The direct chemical reaction/turbulence interaction mentioned above can be considered as one of the nonordinary shear-generated turbulence. In order to elucidate the mechanism of the nonordinary shear-generated turbulence completely, further experiments near the flame zone are necessary.

## Conclusions

Measurements of the correlation of fluctuating velocities parallel and normal to the freestreamwise direction, as well as the mean velocity and turbulent intensity of velocity fluctuation parallel to the freestreamwise direction, in the flat-plate turbulent boundary layer with mass addition and a diffusion flame have been made using a laser Doppler velocimeter.

The profile of the correlation has two maxima and one minimum and becomes negative in the airstream side of the flame zone. This typical profile of the correlation qualitatively corresponds to the profile of the mean velocity gradient normal to the freestream direction in the whole region of the boundary layer near the leading flame edge. In the downstream region, the profile of the correlation does not correspond to the profile of the mean velocity gradient at the flame zone.

These results suggest that the correlation can be treated as that of an ordinary shear-generated turbulence in the whole region of the boundary layer except near the flame zone. Near the flame zone, the correlation is influenced by not only an ordinary shear-generated turbulence but also a nonordinary shear-generated turbulence.

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